

Digital Fundamentals

CHAPTER 5
Boolean Algebra and Logic Simplification

Floyd - Digital Fundamentals, 9th Ed, Chp 4
Modified for Xceli2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 1



Boolean Operations and Expressions

Floyd - Digital Fundamentals, 9th Ed, Chp 4
Modified for Xceli2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 2

Boolean Operations and Expressions

<ul style="list-style-type: none"> • OR (+) $0 + 0 = 0$ $0 + 1 = 1$ $1 + 0 = 1$ $1 + 1 = 1$ 	<ul style="list-style-type: none"> • AND (*) $0 * 0 = 0$ $0 * 1 = 0$ $1 * 0 = 0$ $1 * 1 = 1$ 
--	---

Floyd - Digital Fundamentals, 9th Ed, Chp 4
Modified for Xceli2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 3

Laws and Rules of Boolean Algebra

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xnet2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 4

Laws Boolean Algebra

- Commutative Laws
- Associative Laws
- Distributive Law

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xnet2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 5

Laws Boolean Algebra

Laws	
1	$A + B = B + A$ $AB = BA$
2	$A + (B + C) = (A + B) + C$ $A(BC) = (AB)C$
3	$A(B + C) = AB + AC$ $(A + B)(C + D) = AC + AD + BC + BD$

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xnet2 - Digital Electronics for VHDL, Chp 5


Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 6

Laws of Boolean Algebra

- **Commutative Law of Addition:**

$$A + B = B + A$$



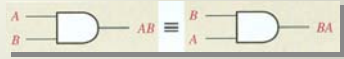
Floyd - Digital Fundamentals, 9th Ed, Chap 4
 Modified for Xnet2 - Digital Electronics for VHDL, Chap 5
 Copyright ©2008 by Pearson Education, Inc.
 Upper Saddle River, New Jersey 07458
 All rights reserved.

Slide 7

Laws of Boolean Algebra

- **Commutative Law of Multiplication:**

$$A * B = B * A$$



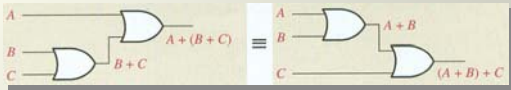
Floyd - Digital Fundamentals, 9th Ed, Chap 4
 Modified for Xnet2 - Digital Electronics for VHDL, Chap 5
 Copyright ©2008 by Pearson Education, Inc.
 Upper Saddle River, New Jersey 07458
 All rights reserved.

Slide 8

Laws of Boolean Algebra

- **Associative Law of Addition:**

$$A + (B + C) = (A + B) + C$$



Floyd - Digital Fundamentals, 9th Ed, Chap 4
 Modified for Xnet2 - Digital Electronics for VHDL, Chap 5
 Copyright ©2008 by Pearson Education, Inc.
 Upper Saddle River, New Jersey 07458
 All rights reserved.

Slide 9

Laws of Boolean Algebra

- Associative Law of Multiplication:

$$A * (B * C) = (A * B) * C$$

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xceli2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 10

Laws of Boolean Algebra

- Distributive Law:

$$A(B + C) = AB + AC$$

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xceli2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 11

Rules of Boolean Algebra

Rules	
1	$A \cdot 0 = 0$
2	$A \cdot 1 = A$
3	$A + 0 = A$
4	$A + 1 = 1$
5	$A \cdot A = A$
6	$A + A = A$
7	$A \cdot \bar{A} = 0$
8	$A + \bar{A} = 1$
9	$\bar{\bar{A}} = A$
10 (a)	$A + \bar{A}B = A + B$
(b)	$\bar{A} + AB = \bar{A} + B$

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xceli2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 12

Rules of Boolean Algebra

- Rule 1

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

Floyd - Digital Fundamentals, 9th Ed. Chap. 4
Modified for Xcel - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 13

Rules of Boolean Algebra

- Rule 2

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

Floyd - Digital Fundamentals, 9th Ed. Chap. 4
Modified for Xcel - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 14

Rules of Boolean Algebra

- Rule 3

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Floyd - Digital Fundamentals, 9th Ed. Chap. 4
Modified for Xcel - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 15

Rules of Boolean Algebra

- Rule 4

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Floyd - Digital Fundamentals, 9th Ed. 4
Modified for Xcel - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 16

Rules of Boolean Algebra

- Rule 5

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

Floyd - Digital Fundamentals, 9th Ed. 4
Modified for Xcel - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 17

Rules of Boolean Algebra

- Rule 6

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Floyd - Digital Fundamentals, 9th Ed. 4
Modified for Xcel - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 18

Rules of Boolean Algebra

- Rule 7

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

Floyd - Digital Fundamentals, 9th Ed. Chp 4
Modified for Xcel - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 19

Rules of Boolean Algebra

- Rule 8

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Floyd - Digital Fundamentals, 9th Ed. Chp 4
Modified for Xcel - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 20

Rules of Boolean Algebra

- Rule 9

$\bar{\bar{A}} = A$

Floyd - Digital Fundamentals, 9th Ed. Chp 4
Modified for Xcel - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 21

Rules of Boolean Algebra

- Rule 10a: $A + \overline{AB} = A + B$

A	B	AB	\overline{AB}	$A + \overline{AB}$	$A + B$
0	0	0	1	0	0
0	1	0	1	0	1
1	0	0	1	1	1
1	1	1	0	1	1

↑ equal ↑

A	B	X	A	B	X
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	1	0	1
1	1	1	1	1	1

AND Truth Table OR Truth Table

Floyd - Digital Fundamentals, 9th Ed. 4
Modified for K102 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 22

Rules of Boolean Algebra

- Rule 10b: $\overline{A} + AB = \overline{A} + B$

A	B	\overline{A}	AB	$\overline{A} + AB$	$\overline{A} + B$
0	0	1	0	1	1
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	1	1

A	B	X	A	B	X
0	0	0	0	0	0
0	1	0	0	1	1
1	0	1	1	0	1
1	1	1	1	1	1

AND Truth Table OR Truth Table

Floyd - Digital Fundamentals, 9th Ed. 4
Modified for K102 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 23

Example

$$X = (A + B)\overline{B}$$

Floyd - Digital Fundamentals, 9th Ed. 4
Modified for K102 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 24

Example

$$Y = A(A + B)$$

Floyd - Digital Fundamentals, 9th Ed, Chap 4
Modified for Xnet2 - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 25

Example

$$Z = BC + \overline{BC}$$

Floyd - Digital Fundamentals, 9th Ed, Chap 4
Modified for Xnet2 - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 26

Example

$$Y = A + AB$$

Floyd - Digital Fundamentals, 9th Ed, Chap 4
Modified for Xnet2 - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 27

Example

$$Y = (A + B)(A + C)$$

Floyd - Digital Fundamentals, 9th Ed, Chp 4
Modified for Kx402 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 28

Example

$$X = A(A + \overline{AB})$$

Floyd - Digital Fundamentals, 9th Ed, Chp 4
Modified for Kx402 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 29

Example

$$X = A(A + \overline{AB})$$

Floyd - Digital Fundamentals, 9th Ed, Chp 4
Modified for Kx402 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 30

DeMorgan's Theorem

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xnet2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 31

DeMorgan's Theorems

- Theorem 1**

$$\overline{X \cdot Y} = \overline{X} + \overline{Y}$$

	X	Y	
	0	0	
	0	1	
	1	0	
	1	1	

	X	Y	
	0	0	
	0	1	
	1	0	
	1	1	

Remember:
"Break the bar,
change the sign"

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xnet2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 32

DeMorgan's Theorems

- Theorem 2**

$$\overline{X + Y} = \overline{X} \cdot \overline{Y}$$

	X	Y	
	0	0	
	0	1	
	1	0	
	1	1	

	X	Y	
	0	0	
	0	1	
	1	0	
	1	1	

Remember:
"Break the bar,
change the sign"

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Xnet2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 33

Example

$$Z = \overline{AB} \bullet \overline{B+C}$$

Floyd - Digital Fundamentals, 9th Ed, Chap 4
Modified for Kx402 - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 34

Example

$$X = \overline{A \bullet \overline{B}} + A \bullet \overline{(A+C)}$$

Floyd - Digital Fundamentals, 9th Ed, Chap 4
Modified for Kx402 - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 35

Example

$$Y = \overline{\overline{AB} + \overline{BC}}$$

Floyd - Digital Fundamentals, 9th Ed, Chap 4
Modified for Kx402 - Digital Electronics for VHDL, Chap 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 36

Standard Forms of Boolean Expressions

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Kval2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 37

Standard Forms of Boolean Expressions

- The sum-of-product (SOP) form
Example: $X = AB + CD + EF$

- The product of sum (POS) form
Example: $X = (A + B)(C + D)(E + F)$

Floyd - Digital Fundamentals, 9th Chp 4
Modified for Kval2 - Digital Electronics for VHDL, Chp 5

Copyright ©2008 by Pearson Education, Inc.
Upper Saddle River, New Jersey 07458
All rights reserved.

Slide 38
