

**Digital Fundamentals**

**CHAPTER 5**  
**Boolean Algebra and Logic Simplification**

Floyd - Digital Fundamentals, 9th Ed, Chp 4  
Modified for Xceli2 - Digital Electronics for VHDL, Chp 5

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**Boolean Operations and Expressions**

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**Boolean Operations and Expressions**

<ul style="list-style-type: none"> <li>• OR (+)</li> <li><math>0 + 0 = 0</math></li> <li><math>0 + 1 = 1</math></li> <li><math>1 + 0 = 1</math></li> <li><math>1 + 1 = 1</math></li> </ul> 	<ul style="list-style-type: none"> <li>• AND (*)</li> <li><math>0 * 0 = 0</math></li> <li><math>0 * 1 = 0</math></li> <li><math>1 * 0 = 0</math></li> <li><math>1 * 1 = 1</math></li> </ul> 
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**Laws and Rules of Boolean Algebra**

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**Laws Boolean Algebra**

- Commutative Laws
- Associative Laws
- Distributive Law

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**Laws Boolean Algebra**

<b>Laws</b>	
1	$A + B = B + A$ $AB = BA$
2	$A + (B + C) = (A + B) + C$ $A(BC) = (AB)C$
3	$A(B + C) = AB + AC$ $(A + B)(C + D) = AC + AD + BC + BD$

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**Laws of Boolean Algebra**

- **Commutative Law of Addition:**  

$$A + B = B + A$$

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**Laws of Boolean Algebra**

- **Commutative Law of Multiplication:**  

$$A * B = B * A$$

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**Laws of Boolean Algebra**

- **Associative Law of Addition:**  

$$A + (B + C) = (A + B) + C$$

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### Laws of Boolean Algebra

- Associative Law of Multiplication:  

$$A * (B * C) = (A * B) * C$$

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### Laws of Boolean Algebra

- Distributive Law:  

$$A(B + C) = AB + AC$$

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### Rules of Boolean Algebra

Rules	
1	$A \cdot 0 = 0$
2	$A \cdot 1 = A$
3	$A + 0 = A$
4	$A + 1 = 1$
5	$A \cdot A = A$
6	$A + A = A$
7	$A \cdot \bar{A} = 0$
8	$A + \bar{A} = 1$
9	$\bar{\bar{A}} = A$
10 (a)	$A + \bar{A}B = A + B$
(b)	$\bar{A} + AB = \bar{A} + B$

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### Rules of Boolean Algebra

- Rule 1

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

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### Rules of Boolean Algebra

- Rule 2

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

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### Rules of Boolean Algebra

- Rule 3

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

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### Rules of Boolean Algebra

- Rule 4

$X = A + 1 = 1$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

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### Rules of Boolean Algebra

- Rule 5

$X = A \cdot A = A$

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

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### Rules of Boolean Algebra

- Rule 6

$X = A + A = A$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

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### Rules of Boolean Algebra

- Rule 7

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

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### Rules of Boolean Algebra

- Rule 8

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

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### Rules of Boolean Algebra

- Rule 9

$\bar{\bar{A}} = A$

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### Rules of Boolean Algebra

- Rule 10a:  $A + \overline{AB} = A + B$

A	B	AB	$\overline{AB}$	$A + \overline{AB}$	$A + B$
0	0	0	1	0	0
0	1	0	1	0	1
1	0	0	1	1	1
1	1	1	0	1	1

↑ equal ↑

A	B	X	A	B	X
0	0	0	0	0	0
0	1	0	0	1	1
1	0	1	1	0	1
1	1	1	1	1	1

AND Truth Table OR Truth Table

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### Rules of Boolean Algebra

- Rule 10b:  $\overline{A} + AB = \overline{A} + B$

A	B	$\overline{A}$	AB	$\overline{A} + AB$	$\overline{A} + B$
0	0	1	0	1	1
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	1	1

A	B	X	A	B	X
0	0	0	0	0	0
0	1	0	0	1	1
1	0	1	1	0	1
1	1	1	1	1	1

AND Truth Table OR Truth Table

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### Example

$$X = (A + B)\overline{B}$$

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**Example**

$$Y = A(A + B)$$

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**Example**

$$Z = BC + \overline{BC}$$

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**Example**

$$Y = A + AB$$

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**Example**

$$Y = (A + B)(A + C)$$

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**Example**

$$X = A(A + \overline{AB})$$

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**Example**

$$X = A(A + \overline{AB})$$

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## DeMorgan's Theorem

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## DeMorgan's Theorems

- Theorem 1**

$$\overline{X \cdot Y} = \overline{X} + \overline{Y}$$

	X	Y	
	0	0	
	0	1	
	1	0	
	1	1	

	X	Y	
	0	0	
	0	1	
	1	0	
	1	1	

Remember:  
"Break the bar,  
change the sign"

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## DeMorgan's Theorems

- Theorem 2**

$$\overline{X + Y} = \overline{X} \cdot \overline{Y}$$

	X	Y	
	0	0	
	0	1	
	1	0	
	1	1	

	X	Y	
	0	0	
	0	1	
	1	0	
	1	1	

Remember:  
"Break the bar,  
change the sign"

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**Example**

$$Z = \overline{AB} \bullet \overline{B+C}$$

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**Example**

$$X = \overline{A \bullet \overline{B}} + A \bullet \overline{(A+C)}$$

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**Example**

$$Y = \overline{\overline{AB} + \overline{BC}}$$

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**Standard Forms of Boolean Expressions**

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**Standard Forms of Boolean Expressions**

- The sum-of-product (SOP) form  
Example:  $X = AB + CD + EF$
  
- The product of sum (POS) form  
Example:  $X = (A + B)(C + D)(E + F)$

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