Department of
Electronics, Computer, and Information Technology

ECT 213 Digital Electronics

Lecture 8: Chapter 7

Binary Arithmetic
(Sub, Mult, Div)

2's Complement

Hex Arithmetic

BCD addition

Subtraction of Binary

$0 - 0 =$ 0 borrow 0
$0 - 1 =$ 1 borrow 1
$1 - 0 =$ 1 borrow 0
$1 - 1 =$ 0 borrow 0

$$\begin{array}{r} 10_2 \\ -\ 0_2 \\ \hline 10 \end{array} \qquad \begin{array}{r} {\scriptstyle 0\ 10} \\ \cancel{1}\cancel{0}_2 \\ -\ 1_2 \\ \hline 1 \end{array} \qquad \begin{array}{r} 11_2 \\ -\ 0_2 \\ \hline 11 \end{array} \qquad \begin{array}{r} 11_2 \\ -\ 1_2 \\ \hline 10 \end{array}$$

A_0	B_0	R_0	B_{out0}
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

A_1	B_1	B_{in1}	R_1	B_{out1}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$
\begin{array}{ccc}
B_{in2} & B_{in1} & \\
 & A_1 & A_0 \\
- & B_1 & B_0 \\
\hline
R_2 & R_1 & R_0 \\
 & + & + \\
 & B_{out1} & B_{out0}
\end{array}
$$

$$
\begin{array}{cccccccc}
100_2 & 100_2 & 100_2 & 100_2 & 110_2 & 110_2 & 110_2 & 110_2 \\
-\ 00_2 & -01_2 & -\ 10_2 & -\ 11_2 & -\ 00_2 & -\ 01_2 & -\ 10_2 & -\ 11_2 \\
\hline
\end{array}
$$

Example 1 - 2:

$$\begin{array}{rcr}
 & & \scriptstyle 1\ \ 1 \\
 & & \scriptstyle 0\ \cancel{10}\ \cancel{10}\ 10 \\
8 & & \cancel{1}0\cancel{0}\cancel{0}_2 \\
-\quad 5 & \Rightarrow & -\quad 101_2 \\
\hline
3 & & 11_2
\end{array}$$

$$\begin{array}{rcr}
 & & \scriptstyle 1\ \ 1 \\
\scriptstyle 3\ \ 10 & & \scriptstyle 0\ 10\ 0\ \cancel{10}\ \cancel{10}\ 10 \\
\cancel{4}\cancel{0} & & \cancel{1}\cancel{0}\cancel{1}\cancel{0}\cancel{0}\cancel{0}_2 \\
-\quad 21 & \Rightarrow & -\quad 10101_2 \\
\hline
19 & & 10011_2 \\
 & & \scriptstyle 16\ 8\ 4\ 2\ 1
\end{array}$$

Example 3 - 4:

$$
\begin{array}{r}
 1\;\;1\;1\;10 \\
0\;\cancel{10}\;\cancel{10}\;\cancel{10}\;\cancel{0}\;10 \\
\cancel{1}\cancel{0}\cancel{0}\cancel{0}\cancel{1}\cancel{0}1_2 \\
-\qquad 111_2 \\
\hline
111110_2
\end{array}
$$

$$
\begin{array}{r}
0\;110\quad 0\;1\;1\;10 \\
1\cancel{1}\cancel{0}\cancel{0}1\cancel{1}\cancel{0}\cancel{0}\cancel{0}_2 \\
-\qquad 110001_2 \\
\hline
101100111_2
\end{array}
$$

Two's Complement:

- Most used method for representing binary numbers and in performing binary arithmetic .

$\underline{D_7}D_6D_5D_4D_3D_2D_1D_0$

- For an 8-bit binary value, D_7, called the

sign bit, indicates if the value is

$D_7 = 0$ $D_7 = 1$

a positive or negative number.

Converting from Decimal to 2's Complement

1. Convert the positive decimal value to binary
   a. Make use the proper number of binary digits are use.
   b. Insure that at the sign bit is zero.

2. If decimal value is negative,
   a. Complement each binary digit (one's complement
   b. Add 1 to the one's complement

Example 9: Convert 60 to an 8-bit two's complement value.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
128	64	32	16	8	4	2	1	
0	0	1	1	1	1	0	0	2

Example 10: Convert -60 to an 8-bit two's complement value.

$$
\begin{array}{lcccccccc}
 & & & & & & 1 & 1 & \\
\text{1's comp} & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
 & & & & & & & + & 1 \\
\hline
 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0_2 = -60
\end{array}
$$

Converting 2's complement value to decimal.

1. If sign bit equals 0, due regular binary-to-decimal conversion

2. If sign bit equals 1 (Negative number),
   a. Complement the 2's complement value
   b. Add 1 to complemented value
   c. Due regular binary-to-decimal conversion

Example 11: Convert the following 2's complement value, $\underline{0}1110011_2$, to decimal.

128	64	32	16	8	4	2	1
0	1	1	1	0	0	1	1

$64 + 32 + 16 + 2 + 1 = 115$

Example 12: Convert the following 2's complement value, $\underline{1}1110011_2$, to decimal. $= -13$

$$
\begin{array}{r}
00001100 \\
+1 \\
\hline
00001101_2 \\
8\,4\;2\,1
\end{array}
= 13
$$

2's complement Arithmetic

Note: assume 8-bit

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
128	64	32	16	8	4	2	1	
0	0	0	0	1	0	0	0	8
0	0	0	0	0	1	0	1	5

Example 13:

$$\begin{array}{r} 8 \\ +\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 0\,0\,0\,0\,1\,0\,0\,0 \\ +\ 0\,0\,0\,0\,0\,1\,0\,1 \\ \hline 0\,0\,0\,0\,1\,1\,0\,1_2 \end{array}$$

Example 14:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
128	64	32	16	8	4	2	1	
0	0	1	1	0	0	1	0	50
0	1	0	0	0	0	1	0	66

```
   50
-  66
-----------
  -16
```

```
            1
1 0 1 1 1 1 0 1
            + 1
---------------
1 0 1 1 1 1 1 0  = -66
```

```
CK      1 1 1 1
0 0 0 0 1 1 1 1
            + 1
---------------
0 0 0 1 0 0 0 0₂  = 16
      16 8 4 2 1
```

```
    1 1 1 1 1 1
  0 0 1 1 0 0 1 0
+ 1 0 1 1 1 1 1 0
-----------------
  1 1 1 1 0 0 0 0₂  = -16
```

Example 15:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
128	64	32	16	8	4	2	1	
0	0	0	1	0	0	1	1	19
0	0	0	1	0	1	0	0	20

$$
\begin{array}{r} -\ 19 \\ 20 \\ \hline 1 \end{array}
$$

$$
\begin{array}{r} 1\,1\,1\,0\,1\,1\,0\,0 \\ +\,1 \\ \hline 1\,1\,1\,0\,1\,1\,0\,1 \end{array} = 19
$$

$$
\begin{array}{r} 1\,1\,1\,1\,1\,1\,\,\, \\ 1\,1\,1\,0\,1\,1\,0\,1 \\ 0\,0\,0\,1\,0\,1\,0\,0 \\ \hline 0\,0\,0\,0\,0\,0\,0\,1_2 \end{array}
$$

Example 16:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

- 80
- 40

Example 17:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

- 75
- 75
